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SOFT EQUIVALENT, UNIFORMLY EQUIVALENT AND LIPSCHITZ EQUIVALENT METRICS

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AUTHORS' CONTRIBUTIONS

This work was carried out in collaboration between all authors. Author Jalil designed the study, performed the statistical analysis, wrote the protocol, and wrote the first draft of the manuscript and managed literature searches. Author B.Surendranath Reddy managed the analyses of the study and literature searches and prepared the final draft. All authors read and approved the final manuscript.

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ABSTRACT

In this paper, we introduce soft equivalent metrics with few examples and derive some properties of soft equivalent metrics. We give sufficient condition for soft equivalent metrics. We also define more stronger versions of soft equivalent metrics namely soft uniformly equivalent and soft Lipschitz equivalent metrics and investigate their relation.

Keywords: Soft set; Soft bounded; soft metric; soft open; soft continuous;soft Lipschitz; soft complete.

1 Introduction

The soft set theory introduced by Molodtsov [1] in the year 1999, is a completely new approach for modeling vagueness and uncertainty. This theory is very useful in dealing problems arise in economics, engineering, environment, medical sciences etc. as these problems cannot be dealt with classical methods. While probability theory, fuzzy set theory and rough set theory etc. are dealing with uncertainty, each of these theories have difficulties as pointed out by Molodtsov [1]. There is no need of membership function in soft set theory and hence very convenient and easy to apply in practise. As this area is new, interesting and has many applications, many researchers started working in this area. Researchers like P. K. Maji et al [2], Murat et al.[3],Sadi Bayrramov and

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CigdemGunduz [4], M. Shabbir et al.[5], Sujao Das and Samanta [6],[7], B. Surendranath Reddy and Sayed Jalil [8] etc. have contributed to the development of soft set theory.

In this paper, section 2 gives the required preliminaries. In section 3, we define soft equivalent metrics and discuss some examples of soft equivalent metrics. We also give sufficient conditions for soft equivalent metrics. In section 4, we introduce more stronger equivalent metrics: soft uniformly equivalent and soft Lipschitz equivalent metrics. We prove that soft uniform equivalences preserves soft Cauchy and soft totally bounded and soft Lipschitz preserves soft boundedness in addition.

2 Preliminaries

In this section we recall the required definitions and results on soft sets.

Definition 2.1. ([1]) Let U be the universe and E be the set of parameters. Let $P(U)$ denote the power set of U and A be a non empty subset of E. A pair (F, A) is called a soft set over U where F is given by $F : A \to P(U)$.
In other words, the soft set is parameterized family of subsets of the set U. For $\varepsilon \in A$, $F(\varepsilon)$ may be considered as the set of ε-elements of the set (F, A), or as the set of ε-approximate elements of the soft set.
i.e. (F, A) is given as consisting of collection of approximation: $(F, A) = \{F(\varepsilon)|\, \varepsilon \in A\}$.

Definition 2.2. ([2]) A soft set (F, E) over U is said to be a null soft set denoted by $\tilde{\phi}$ if $F(e) = \phi \quad \forall e \in E$.

Definition 2.3. ([2]) A soft sets (F, E) over U is said to be a absolute soft set denoted by by $\tilde{U}$ if $F(e) = U \quad \forall e \in E$.

Definition 2.4. ([6]) Let X be a non empty set and E be a non empty parameter set then the function $\varepsilon : E \to X$ is said to be soft element of X.
A soft element ε is said to belongs to a soft set (F, A) of X if $\varepsilon(e) \in A(e)$, for all $e \in A$ and is denoted by $\varepsilon \tilde{\in} (F, A)$.

Definition 2.5. ([6]) Let $\mathbb{R}$ be the set of real numbers and $\mathfrak{B}(\mathbb{R})$ the collection of all non empty bounded subsets of $\mathbb{R}$ and A be a set of parameters then the mapping $F : A \to \mathfrak{B}(\mathbb{R})$ is called a soft real set. It is denoted by (F, A). In particular, if (F, A) is singleton soft set then identifying (F, A) with the corresponding soft element, it will be called a soft real number.
We denote soft real numbers by $\tilde{r}, \tilde{s}, \tilde{t}$ and $\bar{r}, \bar{s}, \bar{t}$ will denote a particular type of soft numbers such that $\bar{r}(\lambda) = r$, for all $\lambda \in A$ etc. For example $\bar{0}(\lambda) = 0$ and $\bar{1}(\lambda) = 1$, for all $\lambda \epsilon A$.

Definition 2.6. ([7]) A soft set (P, A) over X is said to be a soft point if there is exactly one $\lambda \in A$, such that $P(\lambda) = x$, for some $x \in X$ and $P(\mu) = \emptyset$, for all $\mu \in A\backslash\{\lambda\}$. It is denoted by P^x_λ.

Definition 2.7. ([7]) A soft point P^x_λ is said to belong to a soft set (F, A) if $\lambda \in A$ and $P(\lambda) = \{x\} \subset F(\lambda)$ and we write $P^x_\lambda \tilde{\in} (F, A)$.

Definition 2.8. ([7]) Two soft points P^x_λ and P^y_μ are said to be equal if $\lambda = \mu$ and $P(\lambda) = P(\mu)$ i.e $x = y$. Thus $P^x_\lambda \neq P^y_\mu$ if and only if $x \neq y$ or $\lambda \neq \mu$.

Let X be an initial universal set and A be a non empty set of parameters. Let $\tilde{X}$ be the absolute soft set. Let $SP(\tilde{X})$ be the collection of all soft points of $\tilde{X}$. Let $\mathbb{R}(A^*)$ denote the set of all non negative soft real numbers.

Definition 2.9. ([7]) A mapping $\tilde{d} : SP(\tilde{X}) \times SP(\tilde{X}) \to \mathbb{R}(A^*)$ is said to be a soft metric on the soft set $\tilde{X}$ if

1. $\tilde{d}(P^x_\lambda, P^y_\mu)\tilde{\geq}\tilde{0}$ for all $P^x_\lambda, P^y_\mu\tilde{\in}\tilde{X}$,
2. $\tilde{d}(P^x_\lambda, P^y_\mu) = \tilde{0}$ if and only if $P^x_\lambda = P^y_\mu$,
3. $\tilde{d}(P^x_\lambda, P^y_\mu) = \tilde{d}(P^y_\mu, P^x_\lambda)$ for all $P^x_\lambda, P^y_\mu\tilde{\in}\tilde{X}$,
4. $\tilde{d}(P^x_\lambda, P^y_\mu)\tilde{\leq}\tilde{d}(P^x_\lambda, P^y_\mu) + \tilde{d}(P^y_\mu, P^z_\gamma)$ for all $P^x_\lambda, P^y_\mu, P^z_\gamma\tilde{\in}\tilde{X}$.

The soft set $\tilde{X}$ with the soft metric $\tilde{d}$ on $\tilde{X}$ is called a soft metric space and denoted by $(\tilde{X}, \tilde{d}, E)$ *or* $(\tilde{X}, \tilde{d})$.

Definition 2.10. ([7]) Let $(\tilde{X}, \tilde{d}, E)$ be a soft metric space and $\tilde{r}$ be a non negative soft real number. Then the set $B(P^x_\lambda, \tilde{r}) = \{P^y_\mu \in SP(\tilde{X})|\tilde{d}(P^x_\lambda, P^y_\mu)\tilde{<}\tilde{r}\}$ is called soft open ball with center P^x_λ and of radius $\tilde{r}$. If $\tilde{d}_1$ and $\tilde{d}_2$ are two soft metrics on $\tilde{X}$, then we denote soft open balls by $B_{\tilde{d}_1}(P^x_\lambda, \tilde{r})$ and $B_{\tilde{d}_2}(P^x_\lambda, \tilde{r})$.

Definition 2.11. ([7]) Let $(\tilde{X}, \tilde{d}, E)$ be soft metric space (Y, A) be a non null soft subset of $\tilde{X}$ Then (Y, A) is said to be soft open in $(\tilde{X}, \tilde{d}, E)$ if, given $P^x_\lambda\tilde{\in}\tilde{Y}$ there exists a soft real number $\tilde{r}\tilde{>}\tilde{0}$ such that $B(P^x_\lambda, \tilde{r})\tilde{\subset}(Y, A)$.

Definition 2.12. ([8]) Let $(\tilde{X}, \tilde{d}, E)$ be soft metric space and (Y, A) be non empty soft subset of $\tilde{X}$ then (Y, A) is soft bounded if there exist a soft real number $\tilde{\epsilon}\tilde{>}\tilde{0}$ and a soft point $P^x_\lambda\tilde{\in}\tilde{Y}$ such that $(Y, A)\tilde{\subset}B(P^x_\lambda, \tilde{\epsilon})$.

Definition 2.13. ([8]) Let $(\tilde{X}, \tilde{d}, E)$ be a soft metric space and $(Y, A)\tilde{\subset}(\tilde{X}, E)$. We say that (Y, A) is soft totally bounded if for a given $\tilde{\epsilon}\tilde{>}\tilde{0}$, there exists an $\tilde{\epsilon}$-net for (Y, A) i.e., there exist finitely many soft points $P^{x_i}_{\lambda_i}\tilde{\in}\tilde{X}$ such that $(Y, A)\tilde{\subset}\tilde{\bigcup}_{i=1}^{n} B(P^{x_i}_{\lambda_i}, \tilde{\epsilon})$.

Definition 2.14. ([7]) Let $(\tilde{X}, \tilde{d}_1, E_1)$ and $(\tilde{Y}, \tilde{d}_2, E_2)$ be two soft metric space. The mapping $(f, \phi) : (\tilde{X}, \tilde{d}_1, E_1) \to (\tilde{Y}, \tilde{d}_2, E_2)$ is soft mapping where $f : X \to Y$ and $\phi : E_1 \to E_2$ are two mappings.

Definition 2.15. ([7]) Let $(\tilde{X}, \tilde{d}_1, E_1)$ and $(\tilde{Y}, \tilde{d}_2, E_2)$ be two soft metric space. The soft mapping $(f, \phi) : (\tilde{X}, \tilde{d}_1, E_1) \to (\tilde{Y}, \tilde{d}_2, E_2)$ is said to be soft continuous at the soft point $P^x_\lambda\tilde{\in}SP(\tilde{X})$, if for every $\tilde{\epsilon}\tilde{>}\tilde{0}$, there exists a $\tilde{\delta}\tilde{>}\tilde{0}$ such that for any soft points $P^x_\lambda, P^y_\mu\tilde{\in}\tilde{X}$ when $\tilde{d}_1(P^x_\lambda, P^y_\mu)\tilde{\leq}\tilde{\delta}$ then $\tilde{d}_2\big((f, \phi)(P^x_\lambda), (f, \phi)(P^y_\mu)\big)\tilde{\leq}\tilde{\epsilon}$

Definition 2.16. ([7]) Let $(\tilde{X}, \tilde{d}_1, E_1)$ and $(\tilde{Y}, \tilde{d}_2, E_2)$ be two soft metric space. The soft mapping $(f, \phi) : (\tilde{X}, \tilde{d}_1, E_1) \to (\tilde{Y}, \tilde{d}_2, E_2)$ is said to be soft uniformly continuous mapping if given any $\tilde{\epsilon}\tilde{>}\tilde{0}$, there exists a $\tilde{\delta}\tilde{>}\tilde{0}$ ($\tilde{\delta}$ depends only on $\tilde{\epsilon}$) whenever $\tilde{d}_1(P^x_\lambda, P^y_\mu)\tilde{\leq}\tilde{\delta}$ then $\tilde{d}_2\big((f, \phi)(P^x_\lambda), (f, \phi)(P^y_\mu)\big)\tilde{\leq}\tilde{\epsilon}$

Definition 2.17. A soft mapping $(f, \phi) : (\tilde{X}, \tilde{d}_1, E) \to (\tilde{Y}, \tilde{d}_2, E)$ is called Lipschitz if there is a soft real number $\tilde{k}$ such that
$\tilde{d}_2\big((f, \phi)(P^x_\lambda), (f, \phi)(P^y_\mu)\big)\tilde{\leq}\tilde{k}\tilde{d}(P^x_\lambda), P^y_\mu)$ for all $P^x_\lambda, P^y_\mu\tilde{\in}\tilde{X}$

Definition 2.18. ([7]) A soft metric space $(\tilde{X}, \tilde{d}, E)$ is called soft complete if every Cauchy soft sequence in $\tilde{X}$ converges to some soft point of $\tilde{X}$.

Definition 2.19. ([3]) A soft metric space $(\tilde{X}, \tilde{d}, E)$ is called soft compact if it is both soft complete and soft totally bounded.

3 Soft Equivalent Metrics

In this section we introduce soft equivalent metrics with examples and prove some of their properties.

Definition 3.1. Two soft metrics $\tilde{d_1}$ and $\tilde{d_2}$ on a non empty set $\tilde{X}$ are said to be soft equivalent if every soft open set G_1 in $(\tilde{X}, \tilde{d_1})$ is soft open in $(\tilde{X}, \tilde{d_2})$ and every soft open set in G_2 in $(\tilde{X}, \tilde{d_2})$ is soft open in $(\tilde{X}, \tilde{d_1})$.

Theorem 3.1. *Let $SM(\tilde{X})$ be the set of all soft metrics on $\tilde{X}$. For $\tilde{d_1}, \tilde{d_2}$ in $SM(\tilde{X})$ we write it as $\tilde{d_1} \sim \tilde{d_2}$ if $\tilde{d_1}$ and $\tilde{d_2}$ are soft equivalent metrics. Then the relation $\sim$ is an equivalence relation on $SM(\tilde{X})$.*

Proof. Reflexive and symmetric are clear by the definition of soft equivalent metrics.
Suppose $\tilde{d_1} \sim \tilde{d_2}$ and $\tilde{d_2} \sim \tilde{d_3}$.
Let G_1 be soft open set in $(\tilde{X}, \tilde{d_1})$. Since $\tilde{d_1} \sim \tilde{d_2}$, G_1 is soft open in $(\tilde{X}, \tilde{d_2})$.
Again as $\tilde{d_2} \sim \tilde{d_3}$, G_1 is soft open in $(\tilde{X}, \tilde{d_3})$.
Therefore every soft open set in $(\tilde{X}, \tilde{d_1})$ is also soft open in $(\tilde{X}, \tilde{d_3})$.
Similarly, every soft open in $(\tilde{X}, \tilde{d_3})$ is also soft open in $(\tilde{X}, \tilde{d_1})$.
Therefore, $\tilde{d_1} \sim \tilde{d_3}$ and hence the relation $\sim$ is transitive. □

Theorem 3.2. *Let $(\tilde{X}, \tilde{d})$ be a soft metric then there exists a soft bounded metric on $\tilde{X}$ which is soft equivalent to $\tilde{d}$.*

Proof. Let $(\tilde{X}, \tilde{d}, E)$ be a soft metric space. Define $\tilde{\rho}$ on $\tilde{X}$ by
$\tilde{\rho}(P_\lambda^x, P_\mu^y) = \min\{\tilde{1}, \tilde{d}(P_\lambda^x, P_\mu^y)\}$ then $\tilde{\rho}$ is a soft metric on $\tilde{X}$.
Since $\tilde{X}\tilde{\subseteq}B_{\tilde{\rho}}(P_\lambda^x, \tilde{\epsilon}) = \{P_\mu^y | \tilde{\rho}(P_\lambda^x, P_\mu^y) < \tilde{r}\}$ for any $P_\lambda^x \tilde{\in} \tilde{X}$ and $\tilde{r}\tilde{>}\tilde{1}$, $\tilde{X}$ is soft bounded in $\tilde{\rho}$ and hence $\tilde{\rho}$ is soft bounded metric.
To show $\tilde{\rho}$ and $\tilde{d}$ are soft equivalent, Let $\tilde{G}_1$ be a soft open set in $(\tilde{X}, \tilde{d})$, and P_λ^x be a soft point in $\tilde{G}_1$. Then there exist a soft real number $\tilde{\epsilon}$ such that $B_{\tilde{d}}(P_\lambda^x, \tilde{\epsilon})\tilde{\subseteq}\tilde{G}_1$. If this holds for $\tilde{\epsilon}$, then it holds for any $\tilde{0}\tilde{<}\tilde{r}\tilde{<}\tilde{\epsilon}$.
Then $\tilde{\rho}(P_\lambda^x, P_\mu^y)\tilde{<}\tilde{\epsilon} \Leftrightarrow \min\{\tilde{1}, \tilde{d}(P_\lambda^x, P_\mu^y)\}\tilde{<}\tilde{\epsilon} \Leftrightarrow \tilde{d}(P_\lambda^x, P_\mu^y)\tilde{<}\tilde{\epsilon}$.
Therefore $B_{\tilde{\rho}}(P_\lambda^x, \tilde{\epsilon}) = B_{\tilde{d}}(P_\lambda^x, \tilde{\epsilon})\tilde{\subseteq}\tilde{G}_1$. Thus $\tilde{G}_1$ is soft open set in $(\tilde{X}, \tilde{\rho})$.
Now we will prove every open soft set in $(\tilde{X}, \tilde{\rho})$ is open in $(\tilde{X}, \tilde{d})$.Let $\tilde{G}_2$ be a soft open set in $(\tilde{X}, \tilde{\rho})$, then by similar argument, $\tilde{G}_2$ is a soft open set in $(\tilde{X}, \tilde{d})$.
$\Rightarrow$ $\tilde{\rho}$ and $\tilde{d}$ are soft equivalent. □

Theorem 3.3. *Let $(\tilde{X}_1, \tilde{d_1})$ and $(\tilde{X}_1, \tilde{d_2})$ be two soft metric spaces. Then $\tilde{d_1}$ and $\tilde{d_2}$ are soft equivalent if and only if*

1. *For each $P_\lambda^x \tilde{\in} \tilde{X}$ and for each soft open ball $B_{\tilde{d_1}}(P_\lambda^x, \tilde{\epsilon})$, there exists a soft open ball $B_{\tilde{d_2}}(P_\lambda^x, \tilde{\epsilon'})$ contained in $B_{\tilde{d_1}}(P_\lambda^x, \tilde{\epsilon})$ and*

2. *For each $P_\lambda^x \tilde{\in} \tilde{X}$ and for each soft open ball $B_{\tilde{d_2}}(P_\lambda^x, \tilde{\epsilon})$, there exists a soft open ball $B_{\tilde{d_1}}(P_\lambda^x, \tilde{\epsilon'})$ contained in $B_{\tilde{d_2}}(P_\lambda^x, \tilde{\epsilon})$.*

Proof. Assume that $\tilde{d_1}$ and $\tilde{d_2}$ are soft equivalent. Since for each soft point P_λ^x, the soft open ball $B(P_\lambda^x, \tilde{\epsilon})$ is a soft open set, therefore there exists a soft real number $\tilde{\epsilon'}\tilde{>}\tilde{0}$ such that $B_{\tilde{d_2}}(P_\lambda^x, \tilde{\epsilon'})\tilde{\subset}B_{\tilde{d_1}}(P_\lambda^x, \tilde{\epsilon})$.
Similarly 2. can be proved.
Conversely, assume that 1. and 2. hold and we will show that $\tilde{d}$ and $\tilde{d}'$ are soft equivalent.
Let (G, A) be a soft open set in $(\tilde{X}, \tilde{d})$ and P_λ^x be a soft point in (G, A). So there exists a soft real number $\tilde{\epsilon}\tilde{>}\tilde{0}$ such that $B_{\tilde{d_1}}(P_\lambda^x, \tilde{\epsilon})\tilde{\subset}(G, A)$
By assumption 1., there is a soft real number $\tilde{\epsilon'}\tilde{>}\tilde{0}$ such that $B_{\tilde{d_2}}(P_\lambda^x, \tilde{\epsilon'})\tilde{\subset}B_{\tilde{d_1}}(P_\lambda^x, \tilde{\epsilon})$
$\Rightarrow$ $B_{\tilde{d_2}}(P_\lambda^x, \tilde{\epsilon'})\tilde{\subset}(G, A)$

Thus, (G, A) is a soft open set in $(\tilde{X}, \tilde{d_2})$.
Similarly, using the assumption 2., it follows that any soft open set in $(\tilde{X}, \tilde{d_2})$ is soft open in $(\tilde{X}, \tilde{d_1})$.
Thus, $\tilde{d_1}$ and $\tilde{d_2}$ are soft equivalent.

□

Example 3.4. *Let $(\tilde{X}, \tilde{d})$ be a soft metric space, and $\tilde{\gamma}$ be defined as $\tilde{\gamma}(P_\lambda^x, P_\mu^y) = \frac{\tilde{d}(P_\lambda^x, P_\mu^y)}{\tilde{1}+\tilde{d}(P_\lambda^x, P_\mu^y)}$, for all $P_\lambda^x, P_\mu^y \tilde{\in} \tilde{X}$ then $\tilde{d}$ and $\tilde{\gamma}$ are soft equivalent.*

Solution 3.5. Let P_λ^x be any soft point in $\tilde{X}$ and P_μ^y be any soft point in $B_{\tilde{d}}(P_\lambda^x, \tilde{\epsilon})$ then
$P_\mu^y \tilde{\in} B_{\tilde{d}}(P_\lambda^x, \tilde{\epsilon})$ if and only if $\tilde{d}(P_\lambda^x, P_\mu^y) \tilde{<} \tilde{\epsilon}$
if and only if $\frac{\tilde{d}(P_\lambda^x, P_\mu^y)}{\tilde{1}+\tilde{d}(P_\lambda^x, P_\mu^y)} \tilde{<} \frac{\tilde{\epsilon}}{\tilde{1}+\tilde{\epsilon}}$
if and only if $\tilde{\gamma}(P_\lambda^x, P_\mu^y) \tilde{<} \frac{\tilde{\epsilon}}{\tilde{1}+\tilde{\epsilon}}$
if and only if $B_{\tilde{\gamma}}(P_\lambda^x, \frac{\tilde{\epsilon}}{\tilde{1}+\tilde{\epsilon}}) = B_{\tilde{d}}(P_\lambda^x, \tilde{\epsilon})$
For a soft real number $\tilde{\epsilon}$ such that $\tilde{0} \tilde{<} \tilde{\epsilon} \tilde{<} \tilde{1}$, a similar calculation will show that,
$B_{\tilde{\gamma}}(P_\lambda^x, \tilde{\epsilon}) = B_{\tilde{d}}(P_\lambda^x, \frac{\tilde{\epsilon}}{\tilde{1}-\tilde{\epsilon}})$
Hence $\tilde{d}$ and $\tilde{\gamma}$ are soft equivalent.

Remark 3.1. By Theorem 3.4, and Example 3.5, $\tilde{d}$, $\tilde{\rho}$ and $\tilde{\gamma}$ are soft equivalent.

Theorem 3.6. *Two soft metrics $\tilde{d}$ and $\tilde{d}^*$ on a $SP(\tilde{X})$, are soft equivalent if*
$\tilde{s}\tilde{d}(P_\lambda^x, P_\mu^y) \tilde{\leq} \tilde{d}^*(P_\lambda^x, P_\mu^y) \tilde{\leq} \tilde{t}\tilde{d}(P_\lambda^x, P_\mu^y)$ for all $P_\lambda^x, P_\mu^y \tilde{\in} \tilde{X}$ and for some soft real numbers $\tilde{s}$ and $\tilde{t}$.

Proof. Suppose that the soft real numbers $\tilde{s}$ and $\tilde{t}$ exist such that,
$\tilde{s}\tilde{d}(P_\lambda^x, P_\mu^y) \tilde{\leq} \tilde{d}^*(P_\lambda^x, P_\mu^y) \tilde{\leq} \tilde{t}\tilde{d}(P_\lambda^x, P_\mu^y)$
Then for any, $P_\lambda^x \in \tilde{X}$, and a soft real number $\tilde{\epsilon} \tilde{>} \tilde{0}$,
$B_{\tilde{d}}(P_\lambda^x, \tilde{\epsilon}) \tilde{\subset} B_{\tilde{d}^*}(P_\lambda^x, \tilde{t}\tilde{\epsilon})$
To prove this let $P_\mu^y \tilde{\in} B_{\tilde{d}}(P_\lambda^x, \tilde{\epsilon}) \Rightarrow \tilde{d}(P_\lambda^x, P_\mu^y) \tilde{<} \tilde{\epsilon}$
$\Rightarrow \tilde{t}\tilde{d}(P_\lambda^x, P_\mu^y) \tilde{<} \tilde{t}\tilde{\epsilon}$
$\Rightarrow \tilde{d}^*(P_\lambda^x, P_\mu^y) \tilde{<} \tilde{t}\tilde{\epsilon} \Rightarrow P_\mu^y \tilde{\in} B_{\tilde{d}^*}(P_\lambda^x, \tilde{t}\tilde{\epsilon})$
similarly, again for any, $P_\mu^y \in \tilde{X}$, and a soft real number $\epsilon' \tilde{>} \tilde{0}$,
$B_{\tilde{d}^*}(P_\mu^y, \tilde{t}\tilde{\epsilon'}) \tilde{\subset} B_{\tilde{d}}(P_\mu^y, \frac{\epsilon'}{s})$
$\Rightarrow \tilde{d}$ and $\tilde{d}^*$ are soft equivalent by Theorem 3.4. □

Remark 3.2. However the converse of the above theorem is not true.
Let $\tilde{d}(P_\lambda^x, P_\mu^y) = \mid x - y \mid + \mid \lambda - \mu \mid$
We know that $\tilde{d}$ and $\tilde{\rho}(P_\lambda^x, P_\mu^y)) = \frac{\tilde{d}(P_\lambda^x, P_\mu^y)}{\tilde{1}+\tilde{d}(P_\lambda^x, P_\mu^y)}$ are soft equivalent.
Suppose, if possible, there exists a soft real number $\tilde{t} \tilde{>} \tilde{0}$ such that
$\tilde{d}(P_\lambda^x, P_\mu^y) \tilde{\leq} \tilde{t}\tilde{\rho}(P_\lambda^x, P_\mu^y)$.
Since $\tilde{\rho}(P_\lambda^x, P_\mu^y) \tilde{\leq} \tilde{1} \Rightarrow \tilde{d}(P_\lambda^x, P_\mu^y) \tilde{\leq} \tilde{t}$ for all $P_\lambda^x, P_\mu^y \tilde{\in} \tilde{X}$
Which is not possible as $\tilde{d}(P_\lambda^x, P_\mu^y) = \tilde{2}\tilde{t} \tilde{>} \tilde{t}$

Theorem 3.7. *Two soft metrics $\tilde{d}$ and $\tilde{d}'$ on a $SP(\tilde{X})$, are soft equivalent if and only if*

1. *Every soft sequence $\{P_{\lambda_n}^{x_n}\}_n$ in $\tilde{X}$ converging to P_λ^x in $(\tilde{X}, \tilde{d})$ converges to P_λ^x in $(\tilde{X}, \tilde{d}')$.*
2. *Every soft sequence $\{P_{\lambda_n}^{x_n}\}_n$ in $\tilde{X}$ converging to P_λ^x in $(\tilde{X}, \tilde{d}')$ converges to P_λ^x in $(\tilde{X}, \tilde{d})$.*

Proof. Let us suppose that $\tilde{d}$ and $\tilde{d}'$ are soft equivalent. Let P^x_λ be a soft point in $\tilde{X}$, and $\tilde{\epsilon}\tilde{>}\tilde{0}$ be a soft real number. By Theorem 3.4, there exist a soft real number $\tilde{\delta}\tilde{>}\tilde{0}$ such that
$B_{\tilde{d}}(P^x_\lambda,\tilde{\delta})\tilde{\subset}B_{\tilde{d}'}(P^x_\lambda,\tilde{\epsilon})$
If $\{P^{x_n}_{\lambda_n}\}_n$ converges to $P^x_\lambda\tilde{\in}\tilde{X}$ in $(\tilde{X},\tilde{d})$, then there exists $p=p(\epsilon)$ such that
$\tilde{d}(P^{x_n}_{\lambda_n},P^x_\lambda)\tilde{<}\tilde{\delta} \quad n\geq p$
Thus, we have,
$P^{x_n}_{\lambda_n}\tilde{\in}B_{\tilde{d}}(P^x_\lambda,\tilde{\delta})\tilde{\subset}B_{\tilde{d}'}(P^x_\lambda,\tilde{\epsilon})$ for all $n\geq p$
That is, $\{P^{x_n}_{\lambda_n}\}$ converges to P^x_λ in $(\tilde{X},\tilde{d}')$. The Proof of 2 is similar.
Conversely, we assume the condition 1. and 2. in the theorem, and we will prove that $\tilde{d}$ and $\tilde{d}'$ are soft equivalent. Suppose on contrary they are not same.
By Theorem 3.4, there is a soft point $P^{x_0}_{\lambda_0}\tilde{\in}\tilde{X}$, and a soft real number $\tilde{\epsilon}\tilde{>}\tilde{0}$ such that for all soft real numbers $\tilde{\delta}\tilde{>}\tilde{0}$,
$B_{\tilde{d}'}(P^{x_0}_{\lambda_0},\tilde{\delta})$ is not contained in $B_{\tilde{d}}(P^{x_0}_{\lambda_0},\tilde{\epsilon})$.
In particular, for each n, $B_{\tilde{d}'}(P^{x_0}_{\lambda_0},\frac{\tilde{1}}{n})$ is not contained in $B_{\tilde{d}}(P^{x_0}_{\lambda_0},\tilde{\epsilon})$
We can select a soft point $P^{x_n}_{\lambda_n}\tilde{\in}B_{\tilde{d}'}(P^{x_n}_{\lambda_n},\frac{\tilde{1}}{n})$ such that P^x_λ is not in $B_{\tilde{d}}(P^{x_0}_{\lambda_0},\tilde{\epsilon})$
This shows that the soft sequence $\{P^{x_n}_{\lambda_n}\}_n$ converges to $P^{x_0}_{\lambda_0}$ in, $(\tilde{X},\tilde{d}')$, but does not converge to $P^{x_0}_{\lambda_0}$ in $(\tilde{X},\tilde{d})$, which contradicts the assumption 2. and hence the proof. □

We prove an important result which gives sufficient condition for soft equivalent metrics.

Theorem 3.8. *Let $(\tilde{X},\tilde{d})$ be a soft metric. Let $\tilde{f}:R(A^*)\to R(A^*)$ be a soft continuous map with the following properties.*

1. $\tilde{f}(\tilde{t})=\tilde{0} \quad \Leftrightarrow \quad \tilde{t}=\tilde{0}$
2. *$\tilde{f}$ is non decreasing i.e. $\tilde{f}(\tilde{x})\tilde{\leq}\tilde{f}(\tilde{y})$ where $\tilde{x}\tilde{\leq}\tilde{y}$*
3. *$\tilde{f}$ is subadditive i.e. $\tilde{f}(\tilde{x}+\tilde{y})\tilde{\leq}\tilde{f}(\tilde{x})+\tilde{f}(\tilde{y})$*
   Then $\tilde{f}\circ\tilde{d}$ is a soft metric on $\tilde{X}$. Moreover the metrics $\tilde{d}$ and $\tilde{f}o\tilde{d}$ are equivalent.

Proof. Firstly, we will show that $(\tilde{f}\circ\tilde{d})$ is a soft metric.
Since $\tilde{d}(P^x_\lambda,P^y_\mu)\tilde{\geq}\tilde{0}$, $\quad (\tilde{f}\circ\tilde{d})(P^x_\lambda,P^y_\mu)\tilde{\geq}\tilde{0}$, for all $P^x_\lambda,P^y_\mu\tilde{\in}\tilde{X}$
$(\tilde{f}\circ\tilde{d})(P^x_\lambda,P^y_\mu)=\tilde{0}$ if and only if $\tilde{f}(\tilde{d}(P^x_\lambda,P^y_\mu))=\tilde{0}$
if and only if $\tilde{d}(P^x_\lambda,P^y_\mu)=\tilde{0}$
if and only if $P^x_\lambda=P^y_\mu$
clearly, $(\tilde{f}\circ\tilde{d})(P^x_\lambda,P^y_\mu)=(\tilde{f}\circ\tilde{d})(P^y_\mu,P^x_\lambda)$
Let $P^x_\lambda,P^y_\mu,P^z_\gamma\tilde{\in}\tilde{X}$, then
$\tilde{d}(P^x_\lambda,P^y_\mu)\tilde{\leq}\tilde{d}(P^x_\lambda,P^z_\gamma)+\tilde{d}(P^z_\gamma,P^y_\mu)$
Since $\tilde{f}$ is non decreasing,
$\tilde{f}\big(\tilde{d}(P^x_\lambda,P^y_\mu)\big)\tilde{\leq}\tilde{f}\big(\tilde{d}(P^x_\lambda,P^z_\gamma)+\tilde{d}(P^z_\gamma,P^y_\mu)\big)$
Since $\tilde{f}$ is subadditive,
$\tilde{f}\big(\tilde{d}(P^x_\lambda,P^y_\mu)\big)\tilde{\leq}\tilde{f}\big(\tilde{d}(P^x_\lambda,P^z_\gamma)\big)+\tilde{f}\big(\tilde{d}(P^z_\gamma,P^y_\mu)\big)$
$(\tilde{f}\circ\tilde{d})(P^x_\lambda,P^y_\mu)\tilde{\leq}(\tilde{f}\circ\tilde{d})(P^x_\lambda,P^z_\gamma)+(\tilde{f}\circ\tilde{d})(P^z_\gamma,P^y_\mu)$
Therefore, $\tilde{f}\circ\tilde{d}$ is a soft metric space on $\tilde{X}$
Now we will show that $\tilde{f}\circ\tilde{d}$ and $\tilde{d}$ are soft equivalent.
Let $P^x_\lambda\tilde{\in}\tilde{X}$ and $\tilde{\epsilon}\tilde{>}\tilde{0}$, then $\tilde{f}(\tilde{\epsilon})\tilde{>}\tilde{0}$
Let $P^y_\mu\tilde{\in}B_{\tilde{f}\circ\tilde{d}}(P^x_\lambda,\tilde{f}(\tilde{\epsilon}))$
$\Rightarrow (\tilde{f}\circ\tilde{d})(P^x_\lambda,P^y_\mu)\tilde{<}\tilde{f}(\tilde{\epsilon})$
$\Rightarrow \tilde{f}\big(\tilde{d}(P^x_\lambda,P^y_\mu)\big)\tilde{<}\tilde{f}(\tilde{\epsilon})$
Since $\tilde{f}$ is non decreasing, we get $\tilde{d}(P^x_\lambda,P^y_\mu)\tilde{<}\tilde{\epsilon}$
$\Rightarrow P^y_\mu\tilde{\in}B_{\tilde{d}}(P^x_\lambda,\tilde{\epsilon})$

Therefore, $B_{\tilde{f}\circ\tilde{d}}(P^x_\lambda, \tilde{f}(\tilde{\epsilon}))\tilde{\subset}B_{\tilde{d}}(P^x_\lambda, \tilde{\epsilon})$
Let $\tilde{\delta}$ be a soft real number such that $\tilde{0}\tilde{<}\tilde{\delta}\tilde{<}\tilde{\epsilon}$.
Let $P^y_\mu\tilde{\in}B_{\tilde{d}}\big(P^x_\lambda, \tilde{f}^{-1}(\tilde{\delta})\big)$
$\Rightarrow\ \tilde{d}(P^x_\lambda, P^y_\mu)\tilde{<}\tilde{f}^{-1}(\tilde{\delta})$
$\Rightarrow\ \tilde{f}\big(\tilde{d}(P^x_\lambda, P^y_\mu)\big)\tilde{<}\tilde{\delta}$
$\Rightarrow\ \tilde{f}\circ\tilde{d}(P^x_\lambda, P^y_\mu)\tilde{<}\tilde{\delta}\tilde{\epsilon}$
$\Rightarrow\ P^y_\mu\tilde{\in}B_{\tilde{f}\circ\tilde{d}}(P^x_\lambda, \epsilon)$
Therefore, $B_{\tilde{d}}\big(P^x_\lambda, \tilde{f}^{-1}(\tilde{\delta})\big)\tilde{\subset}B_{\tilde{f}\circ\tilde{d}}(P^x_\lambda, \epsilon)$
Hence by Theorem 3.4, the soft metrics $\tilde{f}\circ\tilde{d}$ and $\tilde{d}$ are soft equivalent. □

4 Soft Uniform and Lipschitz Equivalence

Let d_1 and d_2 are two soft metrics on $\tilde{X}$. Let $(f,\phi) : (\tilde{X}, \tilde{d}_1) \to (\tilde{X}, \tilde{d}_2)$ is identity soft map and $(f,\phi)^{-1} : (\tilde{X}, \tilde{d}_2) \to (\tilde{X}, \tilde{d}_1)$ be its inverse soft mappings. Using Theorem 3.4, we rewrite the definition of soft equivalent metrics in terms of the soft mapping.

Definition 4.1. Two soft metrics $\tilde{d}_1$ and $\tilde{d}_2$ are said to be soft equivalent if both (f,ϕ) and $(f,\phi)^{-1}$ are soft continuous.

Definition 4.2. Two soft metrics $\tilde{d}_1$ and $\tilde{d}_2$ are said to be Soft uniformly equivalent if both (f,ϕ) and $(f,\phi)^{-1}$ are soft uniformly continuous.

Definition 4.3. Two soft metrics $\tilde{d}_1$ and $\tilde{d}_2$ are said to be soft Lipschitz equivalence (Strongly soft equivalent) if both (f,ϕ) and $(f,\phi)^{-1}$ are soft Lipschitz maps.

Remark 4.1. From the definition, it is clear that Strongly soft equivalent metric implies soft uniformly equivalent metric implies soft equivalent metric space. But the reverse implications are not true. For this,
Let $\tilde{d}_1(P^x_\lambda, P^y_\mu) = \mid x-y \mid + \mid \lambda-\mu \mid$ and $\tilde{d}_2 P^x_\lambda, (P^y_\mu) = \sqrt{\mid x-y \mid + \mid \lambda-\mu \mid}$ and $X = [0,1]$ then $\tilde{d}_1$ and $\tilde{d}_2$ are soft equivalent metric. In fact they are soft uniformly equivalent. But they are not strongly soft equivalent because we cant find $\tilde{c}\tilde{>}\tilde{0}$ such that $\tilde{c}\sqrt{\mid x-y \mid}\tilde{\leq} \mid x-y \mid$ for all $x, y \in X$

Example 4.1. *Let $(\tilde{X}, \tilde{d})$ be a soft metrics. Then the soft metric $\tilde{\rho}$ defined in Example 3.5 is soft uniformly equivalent to $\tilde{d}$.*

Theorem 4.2. *Suppose $(\tilde{X}, \tilde{d}_1)$ is soft compact metric space and $\tilde{d}_2$ is another soft metric on $\tilde{X}$. Then $\tilde{d}_1$ and $\tilde{d}_2$ are soft equivalent if and only if they are soft uniformly equivalent.*

Proof. Let $\tilde{d}_1$ and $\tilde{d}_2$ are soft equivalent then both (f,ϕ) and $(f,\phi)^{-1}$ are soft continuous. Since $(\tilde{X}, \tilde{d}_1)$ is soft compact metric space, (f,ϕ) is soft uniformly continuous.
Since $(f,\phi)^{-1}$ is onto, $(f,\phi)^{-1}(\tilde{X}, \tilde{d}_2) = (\tilde{X}, \tilde{d}_1)$ is soft compact.
$\Rightarrow\ (f,\phi)(f,\phi)^{-1}(\tilde{X}, \tilde{d}_2)$ is soft compact.
$\Rightarrow\ (\tilde{X}, \tilde{d}_2)$ is soft compact.
$\Rightarrow\ (f,\phi)^{-1}$ is soft uniformly continuous.
$\Rightarrow\ \tilde{d}_1$ and $\tilde{d}_2$ are soft uniformly equivalent. □

Theorem 4.3. *Soft uniformly equivalent metrics preserve soft Cauchy sequences.*

Proof. Let $\tilde{d}_1$ and $\tilde{d}_2$ be soft uniformly equivalent metrics.
Then the maps (f,ϕ) and $(f,\phi)^{-1}$ are soft uniformly continuous.
Let $\{P^{x_n}_{\lambda_n}\}$ be soft Cauchy in $(\tilde{X}, \tilde{d}_1)$ and $\tilde{\epsilon}\tilde{>}\tilde{0}$.

Since (f,ϕ) is soft uniformly continuous, there exists $\tilde{\delta}\tilde{>}\tilde{0}$ such that for all P_λ^x, P_μ^y with $\tilde{d}_1(P_\lambda^x, P_\mu^y)\tilde{\leq}\tilde{\delta}$ we have $\tilde{d}_2\big((f,\phi)(P_\lambda^x),(f,\phi)(P_\mu^y)\big)\tilde{\leq}\tilde{\epsilon}$
As $\{P_{\lambda_n}^{x_n}\}$ is Cauchy and $\tilde{\delta}\tilde{>}\tilde{0}$, there exists k such that $\tilde{d}_1(P_{\lambda_n}^{x_n}, P_{\lambda_m}^{x_m})\tilde{\leq}\tilde{\delta}$ for all $n,m\geq k$. $\tilde{d}_2\big((f,\phi)(P_{\lambda_n}^{x_n}, P_{\lambda_m}^{x_m})\big)\tilde{\leq}\tilde{\epsilon}$
Therefore, $\{(f,\phi)(P_{\lambda_n}^{x_n})\}$ is soft Cauchy in $(\tilde{X},\tilde{d}_2)$.
Similarly by using $(f,\phi)^{-1}$ is soft uniformly continuous, we can prove that if $(P_{\lambda_n}^{x_n})$ is soft Cauchy in $(\tilde{X},\tilde{d}_2)$ then $\{(f,\phi)^{-1}(P_{\lambda_n}^{x_n})\}$ is soft Cauchy in $(\tilde{X},\tilde{d}_1)$. □

Theorem 4.4. *Soft uniformly equivalent metrics preserve soft totally bounded sets.*

Proof. Let $\tilde{d}_1$ and $\tilde{d}_2$ be soft uniformly equivalent metrics.
Then the maps (f,ϕ) and $(f,\phi)^{-1}$ are soft uniformly continuous.
Let (Y,A) be soft totally bounded in $(\tilde{X},\tilde{d}_1)$ and $\tilde{\epsilon}\tilde{>}\tilde{0}$.
Since (f,ϕ) is soft uniformly continuous, there exists $\tilde{\delta}\tilde{>}\tilde{0}$ such that for all P_λ^x, P_μ^y with $\tilde{d}_1(P_\lambda^x, P_\mu^y)\tilde{\leq}\tilde{\delta}$ we have $\tilde{d}_2((f,\phi)(P_\lambda^x),(f,\phi)(P_\mu^y))\tilde{\leq}\tilde{\epsilon}$
That is $(f,\phi)(B_{\tilde{d}_1}(P_\lambda^x,\tilde{\delta}))\tilde{\subset}B_{\tilde{d}_2}((f,\phi)(P_\lambda^x,\epsilon))$ for all P_λ^x in $\tilde{X}$.
As (Y,A) is soft totally bounded and $\tilde{\delta}\tilde{>}\tilde{0}$, there exist $P_{\lambda_1}^{x_1},...,P_{\lambda_n}^{x_n}$ in $\tilde{X}$ such that $(Y,A)\tilde{\subset}\tilde{\cup}_{i=1}^n B_{\tilde{d}_1}(P_{\lambda_i}^{x_i},\tilde{\delta})$

$$\begin{aligned}(f,\phi)(\tilde{Y})&\tilde{\subset}(f,\phi)(\tilde{\cup}_{i=1}^n B_{\tilde{d}_1}(P_{\lambda_i}^{x_i},\tilde{\delta}))\\ &\tilde{\subset}\tilde{\cup}_{i=1}^n(f,\phi)(B_{\tilde{d}_1}(P_{\lambda_i}^{x_i},\tilde{\delta}))\\ &\tilde{\subset}\tilde{\cup}_{i=1}^n B_{\tilde{d}_2}((f,\phi)(P_{\lambda_i}^{x_i},\tilde{\delta}))\end{aligned}$$

Therefore, $(f,\phi)(Y,A)$ is soft totally bounded in $(\tilde{X},\tilde{d}_2)$. By using $(f,\phi)^{-1}$ is soft uniformly continuous, we can prove that if (Y,A) is soft totally bounded in $(\tilde{X},\tilde{d}_2)$ then $(f,\phi)^{-1}(Y,A)$ is soft totally bounded in $(\tilde{X},\tilde{d}_1)$. □

Remark 4.2. Soft Uniform equivalence does not preserve soft boundedness. For example, soft usual metric and soft discrete metric on $\tilde{\mathbb{N}}(A)$ are soft uniformly equivalent. $\tilde{\mathbb{N}}(A)$ is bounded in soft discrete metric but not soft bounded in soft usual metric. Where as soft Lipschitz equivalence metrics preserve soft boundedness.

Theorem 4.5. *Soft Lipschitz equivalence preserves soft boundedness.*

Proof. Let $\tilde{d}_1$ and $\tilde{d}_2$ be soft Lipschitz equivalent and (Y,A) be soft bounded in $\tilde{d}_1$.
Since $\tilde{d}_1$ and $\tilde{d}_2$ be soft Lipschitz equivalent, there exists $\tilde{\beta}\tilde{>}\tilde{0}$
such that $\tilde{d}_1(P_\lambda^x, P_\lambda^y)\tilde{\leq}\tilde{\beta}\tilde{d}_1(P_\lambda^x, P_\lambda^y)$ for all $P_\lambda^x, P_\lambda^y\in(Y,A)$.
Since (Y,A) is soft bounded in $\tilde{d}_1$, there exists $\tilde{M}\tilde{>}\tilde{0}$ such that $\tilde{d}_1(P_\lambda^x, P_\lambda^y)\tilde{\leq}\tilde{M}$.
$\Rightarrow \tilde{d}_1(P_\lambda^x, P_\lambda^y)\tilde{\leq}\tilde{\beta}\tilde{M}$. Hence (Y,A) is soft bounded in $\tilde{d}_2$. □

5 Conclusions

In this paper, we defined soft equivalent metrics and discussed some examples of soft equivalence. We also given sufficient condition for soft equivalent metrics. We then defined soft uniform equivalent of metrics and proved that they preserve soft Cauchy sequences and soft totally bounded sets. We also proved soft that Lipschitz equivalent metrics, unlike soft uniform equivalence, preserves soft boundedness.

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Competing Interests

The authors declare that no competing interests exist.

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